

# Strong coupling constants of bottom and charmed mesons with scalar, pseudoscalar and axial vector kaons

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The strong coupling constants,  $g_{D_s DK_0^*}$ ,  $g_{B_s BK_0^*}$ ,  $g_{D_s^* DK}$ ,  $g_{B_s^* BK}$ ,  $g_{D_s^* DK_1}$  and  $g_{B_s^* BK_1}$ , where  $K_0^*$ ,  $K$  and  $K_1$  are scalar, pseudoscalar and axial vector kaon mesons, respectively are calculated in the framework of three-point QCD sum rules. In particular, the correlation functions of the considered vertices when both  $B(D)$  and  $K_0^*(K)(K_1)$  mesons are off-shell are evaluated. In the case of  $K_1$ , which is either  $K_1(1270)$  or  $K_1(1400)$ , the mixing between these two states are also taken into account. A comparison of the obtained result with the existing prediction on  $g_{D_s^* DK}$  as the only coupling constant among the considered vertices, previously calculated in the literature, is also made.

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## I. INTRODUCTION

The strong coupling constants among the bottom and charmed mesons with light scalar, pseudoscalar and axial strange mesons are the main ingredients in analysis of their strong interactions. More accurate determination of these coupling constants is needed to better understand the strong interactions among the participated mesons, construct the strong potentials among them and obtain knowledge about the nature and structure of the encountered particles. Experimentally, it is believed that in the production of the charmonium states like  $J/\psi$  and  $\psi'$  from the  $B_c$  or newly discovered charmonium  $X$ ,  $Y$  and  $Z$  states by the BABAR and BELLE collaborations, there are intermediate two body states containing  $D$ ,  $D_s$ ,  $D^*$  and  $D_s^*$  mesons (for example, the kaon can annihilate the charmonium in a nuclear medium to give  $D$  and  $D_s$  mesons), which decay to the final  $J/\psi$  and  $\psi'$  states exchanging one or more virtual mesons. A similar story would happen in decays of heavy bottomonium. To exactly follow and analyze the procedure in the experiment, we need to have knowledge about the coupling constants among the particles involved.

The strong coupling constants among mesons take place in low energies very far from the perturbative region, where the strong coupling constant between quarks and gluons is large and perturbation theory fails. Hence in the hadronic scale, one should consult to some nonperturbative methods in QCD to describe nonperturbative phenomena. Among the nonperturbative methods, the QCD sum rules approach [1–4] is one of the most powerful, applicable and attractive one as it is based on QCD Lagrangian and is free of a model dependent parameter. This approach has rendered many successful predictions such as its predictions about the vector mesons [5–9]. The three point correlation function has been widely used to calculate many parameters of hadrons (see for instance [10–13]). The QCD sum rules for some strong coupling constants were derived by means of the three point functions in [14]. In the present work, we investigate various strong coupling constants among bottom (charmed)–bottom strange (charmed strange) mesons with scalar, pseudoscalar and axial vector kaons. Calculation of such coupling constants can help us in understanding the nature of the strong interaction among the participating particles.

In the case of the scalar kaon, we consider the  $B_s - B - K_0^*$  and  $D_s - D - K_0^*$  vertices for both  $K_0^*(800)$  and  $K_0^*(1430)$ . Understanding the internal structure of the scalar mesons has been a striking issue in the last 30-40 years. Despite their investigation both theoretically and experimentally, most of their properties are not very clear yet. Detection and identification of the scalar mesons are difficult, experimentally, so the theoretical and phenomenological works can play a crucial role in this regard. In this work, we also calculate the coupling constants  $g_{B_s^*BK}$  and  $g_{D_s^*DK}$  for pseudoscalar  $K$ . The next aim in the present work is to consider the vertices  $B_s^* - B - K_1$  and  $D_s^* - D - K_1$  for both  $K_1(1270)$  and  $K_1(1400)$  axial states taking into account their mixture.

Experimentally, the  $K_1(1270)$  and  $K_1(1400)$  are the mixtures of the strange members of two axial-vector SU(3) octets  $^3P_1(K_1^A)$  and  $^1P_1(K_1^B)$ . To avoid any confusion between the  $B$  meson and the sign  $B$  in the  $K_1^B$ , we will use the  $K_1^{a(b)}$  instead of  $K_1^{A(B)}$  in this article. The  $K_1(1270, 1400)$  are related to the  $K_1^{a,b}$  states via [15, 16]:

$$\begin{aligned} |K_1(1270)\rangle &= |K_1^a\rangle \sin\theta + |K_1^b\rangle \cos\theta, \\ |K_1(1400)\rangle &= |K_1^a\rangle \cos\theta - |K_1^b\rangle \sin\theta, \end{aligned} \tag{1}$$

where the mixing angle  $\theta$  takes the values in the interval  $37^\circ \leq \theta \leq 58^\circ$ ,  $-58^\circ \leq \theta \leq -37^\circ$  [15–19]. The sign ambiguity for the mixing angle is correlated with the fact that one can add arbitrary phase to the  $|K_1^a\rangle$  and  $|K_1^b\rangle$ . Studies on  $B \rightarrow K_1(1270)\gamma$  and  $\tau \rightarrow K_1(1270)\nu_\tau$  lead to the following value for  $\theta$  [20]:

$$\theta = -(34 \pm 13)^\circ \quad (2)$$

In the present work, contributing the quark-quark and quark-gluon condensate diagrams as nonperturbative effects, we evaluate the corresponding correlation functions when both  $B(D)$  and  $K_0^*(K)(K_1)$  mesons are off-shell. Note that recently, we have investigated the  $D_s^*DK^*(892)$ , and  $B_s^*BK^*(892)$  vertices for  $K^*$  being the vector meson in the framework of the three-point QCD sum rules in [21]. Moreover, the following coupling constants have been investigated via three-point and light cone QCD sum rules in the literature:  $D^*D\pi$  [22, 23],  $DD\rho$  [24],  $DDJ/\psi$  [25],  $D^*DJ/\psi$  [26],  $D^*D^*\pi$  [27, 28],  $D^*D^*J/\psi$  [29],  $D_sD^*K$ ,  $D_s^*DK$  [27, 30],  $D_0D_sK$ ,  $D_{s0}DK$  [31],  $DD\omega$  [32],  $D^*D^*\rho$  [33],  $D^*D\rho$  [34],  $B_{s0}BK$ ,  $B_{s1}B^*K$  [35],  $B_{s0}BK$  [36],  $a_0\eta\pi^0$ ,  $a_0\eta'\pi^0$  [37],  $a_0K^+K^-$  [38] and  $f_0K^+K^-$  [38, 39].

The outline of the paper is as follows. In section II, introducing responsible correlation functions, we obtain QCD sum rules for the strong coupling constant of the considered vertices. For each of the scalar, pseudoscalar and axial kaon cases, we will calculate the correlation function when both the  $B(D)$  and  $K_0^*(K)(K_1)$  mesons are off-shell. In the case of the  $K_1$  meson, first we will calculate the QCD sum rules for the vertices  $B_s^* - B - K_1^a$  and  $D_s^* - D - K_1^b$ , then using the relations in Eq. (1), we will acquire the QCD sum rules for the vertices  $B_s^* - B - K_1(1270)$  and  $D_s^* - D - K_1(1400)$ . In obtaining the sum rules for physical quantities, we will consider both light quark-light quark and light quark-gluon condensate diagrams as nonperturbative contributions. Finally, in section III, we numerically analyze the obtained sum rules for the considered strong coupling constants. We will obtain the numerical values for each coupling constant when both the  $B(D)$  and  $K_0^*(K)(K_1)$  states are off-shell. Then taking the average of the two off-shell cases, we will obtain final numerical values for each coupling constant. In this section, we also compare our result on  $g_{D_s^*DK}$  with existing prediction in the literature.

## II. QCD SUM RULES FOR THE STRONG COUPLING CONSTANTS

In this section, we obtain QCD sum rules for the strong coupling constants associated with the  $D_s(B_s) - D(B) - K_0^*$ ,  $D_s^*(B_s^*) - D(B) - K$  and  $D_s^*(B_s^*) - D(B) - K_1$  vertices. We start our discussion considering the sufficient correlation functions responsible for the corresponding strong transition involving each  $K_0^*$ ,  $K$  and  $K_1$  mesons when both  $D(B)$  and  $K_0^*(K)(K_1)$  are off-shell. The following three-point correlation functions describe the considered strong transitions:

- correlation functions corresponding to the  $D_s(B_s) - D(B) - K_0^*$  vertex:

– for  $D(B)$  off-shell:

$$\Pi^{D(B)} = i^2 \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \langle 0 | \mathcal{T} \left( \eta^{K_0^*}(x) \eta^{D(B)}(y) \eta^{D_s(B_s)\dagger}(0) \right) | 0 \rangle, \quad (3)$$

– for  $K_0^*$  off-shell:

$$\Pi^{K_0^*} = i^2 \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \langle 0 | \mathcal{T} \left( \eta^{D(B)}(x) \eta^{K_0^*}(y) \eta^{D_s(B_s)\dagger}(0) \right) | 0 \rangle, \quad (4)$$

- correlation functions corresponding to the  $D_s^*(B_s^*) - D(B) - K$  vertex:

– for  $D(B)$  off-shell:

$$\Pi_\mu^{D(B)} = i^2 \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \langle 0 | \mathcal{T} \left( \eta^K(x) \eta^{D(B)}(y) \eta_\mu^{D_s^*(B_s^*)\dagger}(0) \right) | 0 \rangle, \quad (5)$$

– for  $K$  off-shell:

$$\Pi_\mu^K = i^2 \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \langle 0 | \mathcal{T} \left( \eta^{D(B)}(x) \eta^K(y) \eta_\mu^{D_s^*(B_s^*)\dagger}(0) \right) | 0 \rangle, \quad (6)$$

- correlation functions corresponding to the  $D_s^*(B_s^*) - D(B) - K_1$  vertex:

– for  $D(B)$  off-shell:

$$\Pi_{\mu\nu}^{D(B)} = i^2 \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \langle 0 | \mathcal{T} \left( \eta_\nu^{K_1}(x) \eta^{D(B)}(y) \eta_\mu^{D_s^*(B_s^*)\dagger}(0) \right) | 0 \rangle, \quad (7)$$

$$\Pi_{\mu\nu\nu'}^{D(B)} = i^2 \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \langle 0 | \mathcal{T} \left( \eta_{\nu\nu'}^{K_1}(x) \eta^{D(B)}(y) \eta_\mu^{D_s^*(B_s^*)\dagger}(0) \right) | 0 \rangle, \quad (8)$$

– for  $K_1$  off-shell:

$$\Pi_{\mu\nu}^{K_1} = i^2 \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \langle 0 | \mathcal{T} \left( \eta^{D(B)}(x) \eta_\nu^{K_1}(y) \eta_\mu^{D_s^*(B_s^*)\dagger}(0) \right) | 0 \rangle, \quad (9)$$

$$\Pi_{\mu\nu\nu'}^{K_1} = i^2 \int d^4x d^4y e^{ip' \cdot x} e^{iq \cdot y} \langle 0 | \mathcal{T} \left( \eta^{D(B)}(x) \eta_{\nu\nu'}^{K_1}(y) \eta_\mu^{D_s^*(B_s^*)\dagger}(0) \right) | 0 \rangle, \quad (10)$$

where  $\mathcal{T}$  is the time ordering product,  $q$  is the momentum of the off-shell state,  $p'$  is the momentum of the final on-shell state. We will set the momentum of the initial state as  $p = p' + q$ . In the vertex containing  $K_1$  meson, we have two correlation functions for both off-shell cases since this meson couples into two interpolating currents  $\eta_\nu^{K_1}$  and  $\eta_{\nu\nu'}^{K_1}$ . We will define these couplings in terms of G-parity conserving and G-parity violating decay constants later. The interpolating currents, which produce the considered mesons from the vacuum with the same quantum numbers as the interpolating currents can be written in terms of the quark field operators as following form:

$$\begin{aligned} \eta^{K_0^*}(x) &= \bar{s}(x) U u(x), \\ \eta^K(x) &= \bar{s}(x) \gamma_5 u(x), \\ \eta_\nu^{K_1}(x) &= \bar{s}(x) \gamma_\nu \gamma_5 u(x), \\ \eta_{\nu\nu'}^{K_1}(x) &= \bar{s}(x) \sigma_{\nu\nu'} \gamma_5 u(x), \\ \eta^{D(B)}(x) &= \bar{u}(x) \gamma_5 c(b)(x), \\ \eta^{D_s(B_s)}(x) &= \bar{s}(x) \gamma_5 c(b)(x), \\ \eta_\mu^{D_s^*(B_s^*)}(x) &= \bar{s}(x) \gamma_\mu c(b)(x), \end{aligned} \quad (11)$$

where  $U$  stands for unit matrix and  $u, s, c$  and  $b$  are the up, strange, charm and bottom quark fields, respectively.

According to general philosophy of the QCD sum rules, we calculate the aforementioned correlation functions in two different representations. In physical or phenomenological representation, we calculate them in terms of hadronic parameters. In QCD or theoretical representation, we evaluate them in terms of QCD degrees of freedom like quark masses, quark condensates, etc. with the help of the operator product expansion (OPE), where the perturbative and nonperturbative contributions are separated. The QCD sum rules for strong coupling constants are obtained equating these two different representations through dispersion relation. To suppress contributions of the higher states and continuum, we will apply double Borel transformation with respect to the momentum squared of the initial and final on-shell states to both sides of the obtained sum rules.

First, let us focus on the calculation of the physical sides of the aforesaid correlation functions for example when  $D(B)$  meson is off-shell. Saturating the correlation functions with the complete sets of three participating particles and isolating the ground states and after some straightforward calculations, we obtain:

- physical representation corresponding to the  $D_s(B_s) - D(B) - K_0^*$  vertex:

$$\Pi^{D(B)} = \frac{\langle 0 | \eta^{K_0^*} | K_0^*(p') \rangle \langle 0 | \eta^{D(B)} | D(B)(q) \rangle \langle K_0^*(p') D(B)(q) | D_s(B_s)(p) \rangle \langle D_s(B_s)(p) | \eta^{D_s(B_s)} | 0 \rangle}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s(B_s)}^2)(p'^2 - m_{K_0^*}^2)} + \dots, \quad (12)$$

- physical representation corresponding to the  $D_s^*(B_s^*) - D(B) - K$  vertex:

$$\Pi_\mu^{D(B)} = \frac{\langle 0 | \eta^K | K(p') \rangle \langle 0 | \eta^{D(B)} | D(B)(q) \rangle \langle K(p') D(B)(q) | D_s^*(B_s^*)(p, \epsilon) \rangle \langle D_s^*(B_s^*)(p, \epsilon) | \eta_\mu^{D_s^*(B_s^*)} | 0 \rangle}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s^*(B_s^*)}^2)(p'^2 - m_K^2)} + \dots, \quad (13)$$

- physical representation corresponding to the  $D_s^*(B_s^*) - D(B) - K_1^{a(b)}$  vertex:

$$\begin{aligned} \Pi_{\mu\nu}^{D(B)} &= \frac{\langle 0 | \eta^{D(B)} | D(B)(q) \rangle \langle D_s^*(B_s^*)(p, \epsilon) | \eta_\mu^{D_s^*(B_s^*)} | 0 \rangle}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s^*(B_s^*)}^2)} \\ &\times \left[ \frac{\langle 0 | \eta_\nu^{K_1} | K_1^a(p', \epsilon') \rangle \langle K_1^a(p', \epsilon') D(B)(q) | D_s^*(B_s^*)(p, \epsilon) \rangle}{(p'^2 - m_{K_1^a}^2)} \right. \\ &\left. + \frac{\langle 0 | \eta_\nu^{K_1} | K_1^b(p', \epsilon') \rangle \langle K_1^b(p', \epsilon') D(B)(q) | D_s^*(B_s^*)(p, \epsilon) \rangle}{(p'^2 - m_{K_1^b}^2)} \right] + \dots, \quad (14) \end{aligned}$$

$$\begin{aligned} \Pi_{\mu\nu\nu'}^{D(B)} &= \frac{\langle 0 | \eta^{D(B)} | D(B)(q) \rangle \langle D_s^*(B_s^*)(p, \epsilon) | \eta_\mu^{D_s^*(B_s^*)} | 0 \rangle}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s^*(B_s^*)}^2)} \\ &\times \left[ \frac{\langle 0 | \eta_{\nu\nu'}^{K_1} | K_1^a(p', \epsilon') \rangle \langle K_1^a(p', \epsilon') D(B)(q) | D_s^*(B_s^*)(p, \epsilon) \rangle}{(p'^2 - m_{K_1^a}^2)} \right. \\ &\left. + \frac{\langle 0 | \eta_{\nu\nu'}^{K_1} | K_1^b(p', \epsilon') \rangle \langle K_1^b(p', \epsilon') D(B)(q) | D_s^*(B_s^*)(p, \epsilon) \rangle}{(p'^2 - m_{K_1^b}^2)} \right] + \dots, \quad (15) \end{aligned}$$

where .... represents the contributions of the higher states and continuum, and  $\epsilon$  and  $\epsilon'$  are the polarization vectors associated with the  $D_s^*$  and  $K_1^{a(b)}$  mesons, respectively. From the above equations it is clear that to proceed we need to define the following matrix elements in terms of decay constants as well as strong coupling constants:

$$\begin{aligned}
\langle 0 | \eta^{K_0^*} | K_0^*(p') \rangle &= m_{K_0^*} f_{K_0^*}, \\
\langle 0 | \eta^K | K(p') \rangle &= \frac{m_K^2 f_K}{m_u + m_s}, \\
\langle 0 | \eta^{D(B)} | D(B)(q) \rangle &= \frac{m_{D(B)}^2 f_{D(B)}}{m_{c(b)} + m_u}, \\
\langle 0 | \eta^{D_s(B_s)} | D_s(B_s)(q) \rangle &= \frac{m_{D_s(B_s)}^2 f_{D_s(B_s)}}{m_{c(b)} + m_s}, \\
\langle D_s^*(B_s^*)(p, \epsilon) | \eta_\mu^{D_s^*(B_s^*)} | 0 \rangle &= m_{D_s^*(B_s^*)} f_{D_s^*(B_s^*)} \epsilon_\mu^*, \\
\langle K_0^*(p') D(B)(q) | D_s(B_s)(p) \rangle &= g_{D_s D K_0^*(B_s B K_0^*)} p \cdot p', \\
\langle K(p') D(B)(q) | D_s^*(B_s^*)(p, \epsilon) \rangle &= g_{D_s^* D K(B_s^* B K)} (p' - q) \cdot \epsilon, \\
\langle K_1^{a(b)}(p', \epsilon') D(B)(q) | D_s^*(B_s^*)(p, \epsilon) \rangle &= g_{D_s^* D K_1^{a(b)}(B_s^* B K_1^{a(b)})} \{ m_{D_s^*(B_s^*)}^2 (\epsilon'^* \cdot \epsilon) + (\epsilon'^* \cdot p)(\epsilon \cdot p') \},
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
&\left. \begin{aligned} \langle 0 | \eta_\nu^{K_1} | K_1^a(p', \epsilon') \rangle &= m_{K_1^a} f_{K_1^a} \epsilon'_\nu, \\ \langle 0 | \eta_{\nu\nu'}^{K_1} | K_1^b(p', \epsilon') \rangle &= f_{K_1^{b\perp}} (\epsilon'_\nu p'_{\nu'} - \epsilon'_{\nu'} p'_\nu) \end{aligned} \right\} \text{G-parity conserving definitions,} \\
&\left. \begin{aligned} \langle 0 | \eta_\nu^{K_1} | K_1^a(p', \epsilon') \rangle &= f_{K_1^a} a_0^{\perp, K_1^a} (\epsilon'_\nu p'_{\nu'} - \epsilon'_{\nu'} p'_\nu), \\ \langle 0 | \eta_\nu^{K_1} | K_1^b(p', \epsilon') \rangle &= m_{K_1^b} f_{K_1^{b\perp}} a_0^{\parallel, K_1^b} \epsilon'_\nu \end{aligned} \right\} \text{G-parity violating definitions,}
\end{aligned} \tag{17}$$

where  $f_{K_0^*}$ ,  $f_K$ ,  $f_{D(B)}$ ,  $f_{D_s(B_s)}$  and  $f_{D_s^*(B_s^*)}$  are leptonic decay constants of the  $K_0^*$ ,  $K$ ,  $D(B)$ ,  $D_s(B_s)$  and  $D_s^*(B_s^*)$  mesons, respectively. The  $f_{K_1^a}$  and  $f_{K_1^{b\perp}}$  are decay constants encountered to the calculations from both definitions of the G-parity conserving and violating matrix elements for the axial  $K_1^a$  and  $K_1^b$  states (for more details see [15, 20, 40, 41]). The  $a_0^{\perp, K_1^a}$  and  $a_0^{\parallel, K_1^b}$  are zeroth order Gegenbauer moments. In the above equations, the  $g_{D_s D K_0^*(B_s B K_0^*)}$ ,  $g_{D_s^* D K(B_s^* B K)}$  and  $g_{D_s^* D K_1^{a(b)}(B_s^* B K_1^{a(b)})}$  are strong coupling constants, which we are going to obtain QCD sum rules for them in this section.

Using Eqs. (12)-(16), and summing over the polarization vectors using the

$$\begin{aligned}
\epsilon'_\nu \epsilon_\theta'^* &= -g_{\nu\theta} + \frac{p'_\nu p'_\theta}{m_{K_1^{a(b)}}^2}, \\
\epsilon_\eta \epsilon_\mu^* &= -g_{\eta\mu} + \frac{p_\eta p_\mu}{m_{D_s^*(B_s^*)}^2},
\end{aligned} \tag{18}$$

the final physical representations of the correlation functions for each vertices in the case of  $D(B)$  off-shell is obtained as:

- $D_s(B_s) - D(B) - K_0^*$  vertex:

$$\begin{aligned} \Pi^{D(B)} &= g_{D_s D K_0^*(B_s B K_0^*)}^{D(B)}(q^2) \frac{f_{K_0^*} m_{K_0^*} \frac{f_{D(B)} m_{D(B)}^2}{m_{c(b)} + m_u} \frac{f_{D_s(B_s)} m_{D_s(B_s)}^2}{m_{c(b)} + m_s}}{2 (q^2 - m_{D(B)}^2)(p'^2 - m_{K_0^*}^2)(p^2 - m_{D_s(B_s)}^2)} (m_{D_s(B_s)}^2 + m_{K_0^*}^2 - q^2) \\ &+ \dots, \end{aligned} \quad (19)$$

- $D_s^*(B_s^*) - D(B) - K$  vertex:

$$\begin{aligned} \Pi_\mu^{D(B)} &= g_{D_s^* D K(B_s^* B K)}^{D(B)}(q^2) \frac{f_{D_s^*(B_s^*)} m_{D_s^*(B_s^*)} \frac{f_K m_K^2}{m_s + m_u} \frac{f_{D(B)} m_{D(B)}^2}{m_{c(b)} + m_u}}{(q^2 - m_{D(B)}^2)(p'^2 - m_K^2)(p^2 - m_{D_s^*(B_s^*)}^2)} \left[ \left( 1 + \frac{m_K^2 - q^2}{m_{D_s^*}^2} \right) p_\mu - 2p'_\mu \right] \\ &+ \dots, \end{aligned} \quad (20)$$

- $D_s^*(B_s^*) - D(B) - K_1$  vertex:

$$\begin{aligned} \Pi_{\mu\nu}^{D(B)} &= \left( g_{D_s^* D K_1^a(B_s^* B K_1^a)}^{D(B)}(q^2) \frac{m_{K_1^a} f_{K_1^a}}{(p'^2 - m_{K_1^a}^2)} + g_{D_s^* D K_1^b(B_s^* B K_1^b)}^{D(B)}(q^2) \frac{m_{K_1^b} f_{K_1^b} a_0^{\parallel, K_1^b}}{(p'^2 - m_{K_1^b}^2)} \right) \\ &\times \frac{\frac{f_{D(B)} m_{D(B)}^2}{m_{c(b)} + m_u} f_{D_s^*(B_s^*)} m_{D_s^*(B_s^*)}}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s^*(B_s^*)}^2)} \left\{ m_{D_s^*(B_s^*)}^2 g_{\mu\nu} + \text{other structures} \right\} + \dots, \end{aligned} \quad (21)$$

$$\begin{aligned} \Pi_{\mu\nu\nu'}^{D(B)} &= \left( g_{D_s^* D K_1^a(B_s^* B K_1^a)}^{D(B)}(q^2) \frac{f_{K_1^a} a_0^{\perp, K_1^a}}{(p'^2 - m_{K_1^a}^2)} + g_{D_s^* D K_1^b(B_s^* B K_1^b)}^{D(B)}(q^2) \frac{f_{K_1^b} a_0^{\perp, K_1^b}}{(p'^2 - m_{K_1^b}^2)} \right) \\ &\times \frac{\frac{f_{D(B)} m_{D(B)}^2}{m_{c(b)} + m_u} f_{D_s^*(B_s^*)} m_{D_s^*(B_s^*)}}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s^*(B_s^*)}^2)} \left\{ m_{D_s^*(B_s^*)}^2 (g_{\mu\nu} p'_{\nu'} - g_{\mu\nu'} p'_\nu) + \text{other structures} \right\} + \dots, \end{aligned} \quad (22)$$

where to calculate the coupling constants, we will choose the structures  $p_\mu$  and  $g_{\mu\nu}(g_{\mu\nu} p'_{\nu'} - g_{\mu\nu'} p'_\nu)$  from both sides of the correlation functions corresponding to the vertices containing the  $K$  and  $K_1$  with current  $\eta_\nu^{K_1}$  ( $K_1$  with current  $\eta_{\nu\nu'}^{K_1}$ ), respectively. From the similar way, one can easily obtain the physical representations of the correlation functions associated with the  $K_0^*(K)(K_1)$  off-shell.

Now, we concentrate to calculate the QCD or theoretical sides of the considered correlation functions. The QCD representations of the correlation functions are obtained in deep Euclidean region, where  $p^2 \rightarrow -\infty$  and  $p'^2 \rightarrow -\infty$  via OPE. For this aim, each correlation function in QCD side is written in terms of the perturbative and non-perturbative parts as:

$$\Pi^i = \Pi_{per}^i + \Pi_{nonper}^i \quad (23)$$

where  $i$  stands for  $D(B)$  or  $K_0^*(K)(K_1)$  and the perturbative parts are defined in terms of double dispersion integral as following:

$$\Pi_{per}^i = -\frac{1}{4\pi^2} \int ds' \int ds \frac{\rho^i(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}, \quad (24)$$

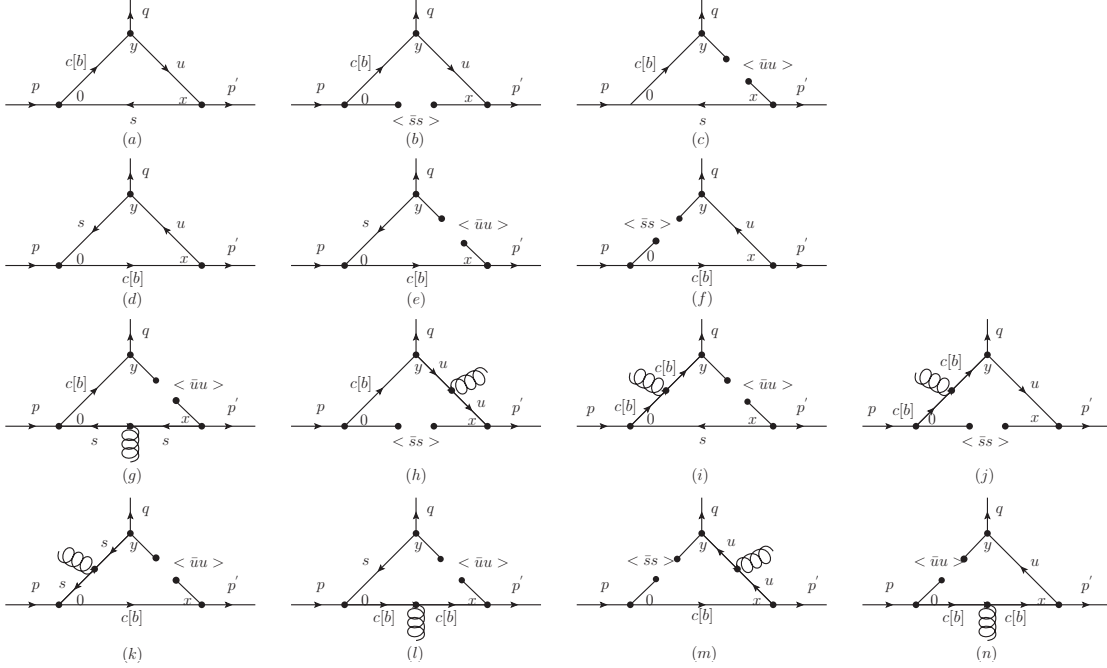


FIG. 1. Diagrams considered in the calculations. The first and third line diagrams refer to the  $B(D)$  off-shell and the second and fourth line diagrams show the case when  $K_0^*(K)(K_1)$  is off-shell.

where  $\rho^i(s, s', q^2)$  are called spectral densities. In order to obtain the spectral density, we need to calculate the bare loop diagrams (a) and (d) in Fig. (1) for  $D(B)$  and  $K_0^*(K)(K_1)$  off-shell, respectively. We calculate these diagrams in terms of the usual Feynman integrals by the help of the Cutkosky rules, where the quark propagators are replaced by Dirac delta function, i.e.,  $\frac{1}{q^2 - m^2} \rightarrow (-2\pi i)\delta(q^2 - m^2)$ . As a result, the spectral densities are obtained as follows:

- $D_s(B_s) - D(B) - K_0^*$  vertex:

- $D(B)$  off-shell:

$$\rho^{D(B)}(s, s', q^2) = \frac{N_c}{2 \lambda^{1/2}(s, s', q^2)} \left\{ m_s \left( m_u(m_s + m_u) - q^2 \right) - s m_u - m_{c(b)} \left( (m_s + m_u)^2 - s' - m_{c(b)}(m_s + m_u) \right) \right\}, \quad (25)$$

- $K_0^*$  off-shell:

$$\rho^{K_0^*}(s, s', q^2) = \frac{N_c}{2 \lambda^{1/2}(s, s', q^2)} \left\{ m_s s' + m_{c(b)} \left( (m_s + m_u)^2 - q^2 \right) - m_{c(b)}^2(m_s + m_u) + m_u(s - m_s(m_s + m_u)) \right\}, \quad (26)$$

- $D_s^*(B_s^*) - D(B) - K$  vertex:

- $D(B)$  off-shell:

$$\rho^{D(B)}(s, s', q^2) = \frac{N_c}{\lambda^{3/2}(s, s', q^2)} \left[ (m_u - m_s)(q^2 - s) \left( m_{c(b)} m_s^2 + m_u(s - m_s^2 - q^2) \right) \right]$$



$$\begin{aligned}
& -s' \left( -m_s^3 m_u + 2m_{c(b)}^3 (m_u - m_s) - 2m_s^2 q^2 + m_{c(b)}^2 (2m_s m_u + q^2 - s) \right. \\
& + q^2 (s - q^2) + m_s m_u (s + q^2) + m_{c(b)} (m_s - m_u) (m_s^2 + q^2 + s) \Big) \\
& \left. - s'^2 (m_{c(b)}^2 - m_{c(b)} m_s + m_{c(b)} m_u + q^2) \right], \tag{27}
\end{aligned}$$

–  $K$  off-shell:

$$\begin{aligned}
\rho^K(s, s', q^2) = \frac{N_c}{\lambda^{3/2}(s, s', q^2)} & \left[ (m_{c(b)} - m_u)(q^2 - s) \left( m_{c(b)}^2 (m_{c(b)} - m_u) + m_u (-m_s m_u - q^2 \right. \right. \\
& + s) \Big) + \left( m_{c(b)}^3 (m_s - m_u) + 2m_s^3 m_u + m_{c(b)}^2 (-m_s m_u - 2q^2) + m_s^2 (q^2 - s) \right. \\
& + q^2 (s - q^2) - m_s m_u (q^2 + s) + m_{c(b)} (-2m_s^3 + 2m_s^2 m_u + m_u (q^2 + s) \\
& \left. \left. + m_s (q^2 + s)) \right) s' + (-m_{c(b)} m_s + m_s^2 + m_s m_u + q^2) s'^2 \right], \tag{28}
\end{aligned}$$

- $D_s^*(B_s^*) - D(B) - K_1$  vertex:

–  $D(B)$  off-shell:

$$\begin{aligned}
\rho_1^{D(B)}(s, s', q^2) = -2N_c I_0(s, s', q^2) & \left[ 2m_s^3 + 2m_s^2 m_u - 2m_{c(b)} m_s (m_s + m_u) \right. \\
& + m_s (s + s' - q^2) + 4A (m_{c(b)} - m_u) \\
& + B (2m_u s + m_{c(b)} (q^2 - s - s') + m_s (-q^2 + 3s + s')) \\
& \left. + C (-2m_{c(b)} s' + m_u (-q^2 + s + s') + m_s (-q^2 + s + 3s')) \right], \tag{29}
\end{aligned}$$

$$\begin{aligned}
\rho_2^{D(B)}(s, s', q^2) = 4N_c I_0(s, s', q^2) & \left[ 2A + (m_{c(b)} m_s - m_s^2) - (B + 2G)s + H(q^2 - s - s') \right. \\
& \left. + C (m_{c(b)} (m_s + m_u) - m_s (m_s + m_u) + q^2 - s - s') \right], \tag{30}
\end{aligned}$$

–  $K_1$  off-shell:

$$\begin{aligned}
\rho_1^{K_1}(s, s', q^2) = 2N_c I_0(s, s', q^2) & \left[ 2m_{c(b)} (m_{c(b)}^2 + m_s m_u - m_{c(b)} (m_s + m_u) + s') \right. \\
& - 4D (m_{c(b)} - m_u) + E (2m_{c(b)} - m_s - m_u) (q^2 - s - s') \\
& \left. + 2F s' (-2m_{c(b)} + m_s + m_u) \right], \tag{31}
\end{aligned}$$

$$\begin{aligned}
\rho_2^{K_1}(s, s', q^2) = 4N_c I_0(s, s', q^2) & \left[ -2D + m_{c(b)}^2 - m_{c(b)} m_s - Es \right. \\
& \left. - F (m_{c(b)} (m_{c(b)} - m_s - m_u) + m_s m_u) \right], \tag{32}
\end{aligned}$$

where  $\rho_1$  and  $\rho_2$  correspond to the currents  $\eta_\nu^{K_1}$  and  $\eta_{\nu\nu'}^{K_1}$ , respectively and

$$\begin{aligned}
A &= \frac{1}{2\Delta} \{ m_s^4 q^2 + m_{c(b)}^4 s' + q^2 s s' + m_{c(b)}^2 (m_s^2 (-q^2 + s - s') + s' (-q^2 - s + s')) \\
& - m_s^2 q^2 (-q^2 + s + s') \}, \\
B &= \frac{1}{\Delta} \{ m_s^2 (q^2 - s + s') - s' (2m_{c(b)}^2 - q^2 - s + s') \},
\end{aligned}$$

$$\begin{aligned}
C &= \frac{1}{\Delta} \{ m_{c(b)}^2 (-q^2 + s + s') + s(q^2 - s + s') - m_s^2 (-q^2 - s + s') \}, \\
D &= \frac{1}{2\Delta} \{ m_{c(b)}^4 q^2 + m_s^4 s' + q^2 s s' - m_{c(b)}^2 q^2 (-q^2 + s + s') + m_s^2 (m_{c(b)}^2 (-q^2 + s - s') \\
&\quad + s'(-q^2 - s + s')) \}, \\
E &= \frac{1}{\Delta} \{ s'(2m_s^2 - q^2 - s + s') - m_{c(b)}^2 (q^2 - s + s') \}, \\
F &= \frac{1}{\lambda(s, s', q^2)} \{ (-m_s^2 + m_{c(b)}^2 + s)(-q^2 + s + s') - 2s(m_{c(b)}^2 + s') \}, \\
G &= \frac{1}{\Delta^2} \{ 3m_{c(b)}^4 s'(q^2 - s - s') + m_s^4 (2q^4 - (s - s')^2 - q^2(s + s')) - s s' (-2q^4 + (s - s')^2 \\
&\quad + q^2(s + s')) + m_s^2 (q^6 - q^4(s + s') + (s - s')^2(s + s') - q^2(s^2 - 6s s' + s'^2)) - 2m_{c(b)}^2 \\
&\quad (s'(q^4 - 2s^2 + q^2(s - 2s') + s s' + s'^2) + m_s^2 (q^4 + s^2 + s s' - 2s'^2 + q^2(-2s + s')))) \}, \\
H &= \frac{1}{\Delta^2} \{ s^2 (q^4 - 2q^2(s - 2s') + (s - s')^2) + m_s^4 (q^4 + 2q^2(2s - s') + (s - s')^2) + m_{c(b)}^4 \\
&\quad (q^4 + s^2 + 4s s' + s'^2 - 2q^2(s + s')) - 2m_s^2 s (-2q^4 + (s - s')^2 + q^2(s + s')) - 2m_{c(b)}^2 \\
&\quad (m_s^2 (q^4 - 2s^2 + q^2(s - s') + s s' + s'^2) + s(q^4 + s^2 + s s' - 2s'^2 + q^2(-2s + s')))) \}, \quad (33)
\end{aligned}$$

and

$$\begin{aligned}
I_0(s, s', q^2) &= \frac{1}{4\lambda^{1/2}(s, s', q^2)} \\
\Delta &= q^4 + (s - s')^2 - 2q^2(s + s') \\
\lambda(a, b, c) &= a^2 + b^2 + c^2 - 2ac - 2bc - 2ab. \quad (34)
\end{aligned}$$

In the spectral densities,  $N_c = 3$  is the color number and we have kept terms linear in  $m_u$ .

Now, we proceed to calculate the nonperturbative contributions in QCD side. We consider the quark-quark and quark-gluon condensate diagrams presented as (b), (c), (e), (f), (g), (h), (i), (j), (k), (l), (m) and (n) in Fig. (1). Contributions of the diagrams (c), (e), (f), (g), (i), (k), (l), (m) and (n) in Fig. (1) are zero since applying double Borel transformation with respect to the both variables  $p^2$  and  $p'^2$  will kill their contributions because of only one variable appearing in the denominator in these cases. Hence, we consider contributions of only diagrams (b), (h) and (j) in Fig. (1). As a result, we obtain:

- $D_s(B_s) - D(B) - K_0^*$  vertex:

–  $D(B)$  off-shell:

$$\begin{aligned}
\Pi_{nonper}^{D(B)} &= \frac{\langle \bar{s}s \rangle}{2} \left\{ \frac{2m_{c(b)}m_u - m_{c(b)}^2 + q^2}{rr'} - \frac{1}{r} - \frac{1}{r'} + \frac{m_0^2(4m_{c(b)}m_u - m_{c(b)}^2 + q^2)}{4r^2r'} \right. \\
&\quad \left. - \frac{m_0^2}{4rr'} - \frac{m_0^2}{4r^2} + \frac{m_0^2(m_{c(b)}^2 - 4m_{c(b)}m_u - q^2)}{4rr'^2} + \frac{m_0^2}{4r^2} + \frac{m_0^2}{4rr'} \right\}, \quad (35)
\end{aligned}$$

–  $K_0^*$  off-shell:

$$\Pi_{nonper}^{K_0^*} = 0, \quad (36)$$

- $D_s^*(B_s^*) - D(B) - K$  vertex:

–  $D(B)$  off-shell:

$$\Pi_{nonper}^{D(B)} = -\langle \bar{s}s \rangle \left\{ \frac{m_u}{rr'} + \frac{m_0^2}{4} \frac{m_u}{r^2 r'} + \frac{m_0^2}{2} \frac{m_u}{r r'^2} \right\}, \quad (37)$$

–  $K$  off-shell:

$$\Pi_{nonper}^K = 0, \quad (38)$$

•  $D_s^*(B_s^*) - D(B) - K_1$  vertex:

–  $D(B)$  off-shell:

$$\begin{aligned} \Pi_{nonper1}^{D(B)} = & -\langle \bar{s}s \rangle \left[ \frac{m_0^2}{24} \left( \frac{m_{c(b)}^2 - q^2}{r^2 r'} + \frac{2}{rr'} + \frac{1}{r^2} + \frac{1}{r'^2} + \frac{m_{c(b)}^2 - q^2}{rr'^2} \right) \right. \\ & \left. + \frac{q^2 + 2m_u m_{c(b)} - m_{c(b)}^2}{2rr'} - \frac{1}{2r} - \frac{1}{r'} \right], \end{aligned} \quad (39)$$

$$\Pi_{nonper2}^{D(B)} = -\langle \bar{s}s \rangle \left[ \frac{m_{c(b)}}{rr'} - \frac{m_0^2}{24} \frac{m_{c(b)}}{rr'^2} \right], \quad (40)$$

–  $K_1$  off-shell:

$$\Pi_{nonper1}^{K_1} = 0, \quad (41)$$

$$\Pi_{nonper2}^{K_1} = 0, \quad (42)$$

where  $r = p^2 - m_{c(b)}^2$  and  $r' = p'^2 - m_u^2$ . The  $\Pi_{nonper1}$  and  $\Pi_{nonper2}$  correspond to also the currents  $\eta_\nu^{K_1}$  and  $\eta_{\nu\nu'}^{K_1}$ , respectively. In this step, we equate the physical side in the case of  $K_0^*$  and the coefficients of the selected structures in physical sides of  $K$  and  $K_1$  to the corresponding QCD sides and obtain QCD sum rules for the considered strong coupling constants. To suppress the contributions of the higher states and continuum, we also apply the double Borel transformation with respect to the variables  $p^2(p^2 \rightarrow M^2)$  and  $p'^2(p'^2 \rightarrow M'^2)$ . Finally, we get the following sum rules for the considered coupling constants:

•  $D_s(B_s) - D(B) - K_0^*$  vertex:

–  $D(B)$  off-shell:

$$\begin{aligned} g_{D_s D K_0^*(B_s B K_0^*)}^{D(B)}(q^2) = & \frac{2(q^2 - m_{D(B)}^2)(m_{c(b)} + m_u)(m_{c(b)} + m_s)}{m_{D_s(B_s)}^2 m_{D(B)}^2 m_{K_0^*} f_{D_s(B_s)} f_{D(B)} f_{K_0^*} (m_{D_s(B_s)}^2 + m_{K_0^*}^2 - q^2)} e^{\frac{m_{D_s(B_s)}^2}{M^2}} \\ & \times e^{\frac{m_{K_0^*}^2}{M'^2}} \left[ -\frac{1}{4\pi^2} \int_{(m_{c(b)} + m_s)^2}^{s_0} ds \int_{(m_s + m_u)^2}^{s'_0} ds' \rho^{D(B)}(s, s', q^2) \theta[1 - (f^{D(B)}(s, s'))^2] e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}} \right. \\ & \left. + \hat{B} \Pi_{nonper}^{D(B)} \right], \end{aligned} \quad (43)$$

–  $K_0^*$  off-shell:

$$g_{D_s^* D K_0^* (B_s B K_0^*)}^{K_0^*}(q^2) = \frac{-2(q^2 - m_{K_0^*}^2)(m_{c(b)} + m_u)(m_{c(b)} + m_s)}{m_{D_s(B_s)}^2 m_{D(B)}^2 m_{K_0^*} f_{D_s(B_s)} f_{D(B)} f_{K_0^*} (m_{D_s(B_s)}^2 + m_{D(B)}^2 - q^2)} e^{\frac{m_{D_s(B_s)}^2}{M^2}} \\ \times e^{\frac{m_{D_s(B_s)}^2}{M'^2}} \left[ -\frac{1}{4\pi^2} \int_{(m_{c(b)}+m_s)^2}^{s_0} ds \int_{(m_{c(b)}+m_u)^2}^{s'_0} ds' \rho^{K_0^*}(s, s', q^2) \theta[1 - (f^{K_0^*}(s, s'))^2] e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}} \right], \quad (44)$$

•  $D_s^*(B_s^*) - D(B) - K$  vertex:

–  $D(B)$  off-shell:

$$g_{D_s^* D K (B_s^* B K)}^{D(B)}(q^2) = \frac{(q^2 - m_{D(B)}^2)(m_{c(b)} + m_u)(m_s + m_u)}{f_{D_s^*(B_s^*)} f_{D(B)} f_K m_{D_s^*(B_s^*)} m_K^2 m_{D(B)}^2 (1 + \frac{m_K^2 - q^2}{m_{D_s^*(B_s^*)}^2})} e^{\frac{m_{D_s^*(B_s^*)}^2}{M^2}} e^{\frac{m_K^2}{M'^2}} \\ \times \left[ -\frac{1}{4\pi^2} \int_{(m_{c(b)}+m_s)^2}^{s_0} ds \int_{(m_s+m_u)^2}^{s'_0} ds' \rho^{D(B)}(s, s', q^2) \theta[1 - (f^{D(B)}(s, s'))^2] e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}} \right. \\ \left. + \hat{B} \Pi_{nonper}^{D(B)} \right] \quad (45)$$

–  $K$  off-shell:

$$g_{D_s^* D K (B_s^* B K)}^K(q^2) = \frac{(q^2 - m_K^2)(m_{c(b)} + m_u)(m_s + m_u)}{f_{D_s^*(B_s^*)} f_{D(B)} f_K m_{D_s^*(B_s^*)} m_K^2 m_{D(B)}^2 (1 + \frac{m_{D(B)}^2 - q^2}{m_{D_s^*(B_s^*)}^2})} e^{\frac{m_{D_s^*(B_s^*)}^2}{M^2}} e^{\frac{m_{D(B)}^2}{M'^2}} \\ \times \left[ -\frac{1}{4\pi^2} \int_{(m_{c(b)}+m_s)^2}^{s_0} ds \int_{(m_{c(b)}+m_u)^2}^{s'_0} ds' \rho^K(s, s', q^2) \theta[1 - (f^K(s, s'))^2] e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}} \right], \quad (46)$$

•  $D_s^*(B_s^*) - D(B) - K_1$  vertex:

–  $D(B)$  off-shell:

$$\left( g_{D_s^* D K_1^a (B_s^* B K_1^a)}^{D(B)}(q^2) m_{K_1^a} f_{K_1^a} e^{\frac{-m_{K_1^a}^2}{M'^2}} + g_{D_s^* D K_1^b (B_s^* B K_1^b)}^{D(B)}(q^2) m_{K_1^b} f_{K_1^b} a_0^{\parallel, K_1^b} e^{\frac{-m_{K_1^b}^2}{M'^2}} \right) \\ = \frac{(q^2 - m_{D(B)}^2)}{f_{D_s^*(B_s^*)} f_{D(B)} \frac{m_{D(B)}^2}{m_{c(b)}+m_u} m_{D_s^*(B_s^*)}^3} e^{\frac{m_{D_s^*(B_s^*)}^2}{M^2}} \\ \times \left[ -\frac{1}{4\pi^2} \int_{(m_{c(b)}+m_s)^2}^{s_0} ds \int_{(m_s+m_u)^2}^{s'_0} ds' \rho_1^{D(B)}(s, s', q^2) \theta[1 - (f^{D(B)}(s, s'))^2] \right. \\ \left. \times e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}} + \hat{B} \Pi_{nonper1}^{D(B)} \right], \quad (47)$$

$$\begin{aligned}
& \left( g_{D_s^* D K_1^a (B_s^* B K_1^a)}^{D(B)}(q^2) f_{K_1^a} a_0^{\perp, K_1^a} e^{\frac{-m_{K_1^a}^2}{M'^2}} + g_{D_s^* D K_1^b (B_s^* B K_1^b)}^{D(B)}(q^2) f_{K_1^b} e^{\frac{-m_{K_1^b}^2}{M'^2}} \right) \\
&= \frac{(q^2 - m_{D(B)}^2)}{f_{D_s^* (B_s^*)} f_{D(B)} \frac{m_{D(B)}^2}{m_{c(b)} + m_u} m_{D_s^* (B_s^*)}^3} e^{\frac{m_{D_s^* (B_s^*)}^2}{M^2}} \\
&\times \left[ -\frac{1}{4 \pi^2} \int_{(m_{c(b)} + m_s)^2}^{s_0} ds \int_{(m_s + m_u)^2}^{s'_0} ds' \rho_2^{D(B)}(s, s', q^2) \theta[1 - (f^{D(B)}(s, s'))^2] \right. \\
&\times \left. e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}} + \widehat{B} \Pi_{nonper2}^{D(B)} \right], \tag{48}
\end{aligned}$$

–  $K_1$  off-shell:

$$\begin{aligned}
& \left( g_{D_s^* D K_1^a (B_s^* B K_1^a)}^{K_1^a}(q^2) \frac{m_{K_1^a} f_{K_1^a}}{(q^2 - m_{K_1^a}^2)} + g_{D_s^* D K_1^b (B_s^* B K_1^b)}^{K_1^b}(q^2) \frac{m_{K_1^b} f_{K_1^b} a_0^{\parallel, K_1^b}}{(q^2 - m_{K_1^b}^2)} \right) \\
&= \frac{1}{f_{D_s^* (B_s^*)} f_{D(B)} \frac{m_{D(B)}^2}{m_{c(b)} + m_u} m_{D_s^* (B_s^*)}^3} e^{\frac{m_{D_s^* (B_s^*)}^2}{M^2}} e^{\frac{m_{D(B)}^2}{M'^2}} \\
&\times \left[ -\frac{1}{4 \pi^2} \int_{(m_{c(b)} + m_s)^2}^{s_0} ds \int_{(m_{c(b)} + m_u)^2}^{s'_0} ds' \rho_1^{K_1}(s, s', q^2) \theta[1 - (f^{K_1}(s, s'))^2] \right. \\
&\times \left. e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}} + \widehat{B} \Pi_{nonper1}^{K_1} \right], \tag{49}
\end{aligned}$$

$$\begin{aligned}
& \left( g_{D_s^* D K_1^a (B_s^* B K_1^a)}^{K_1^a}(q^2) \frac{f_{K_1^a} a_0^{\perp, K_1^a}}{(q^2 - m_{K_1^a}^2)} + g_{D_s^* D K_1^b (B_s^* B K_1^b)}^{K_1^b}(q^2) \frac{f_{K_1^b}}{(q^2 - m_{K_1^b}^2)} \right) \\
&= \frac{1}{f_{D_s^* (B_s^*)} f_{D(B)} \frac{m_{D(B)}^2}{m_{c(b)} + m_u} m_{D_s^* (B_s^*)}^3} e^{\frac{m_{D_s^* (B_s^*)}^2}{M^2}} e^{\frac{m_{D(B)}^2}{M'^2}} \\
&\times \left[ -\frac{1}{4 \pi^2} \int_{(m_{c(b)} + m_s)^2}^{s_0} ds \int_{(m_{c(b)} + m_u)^2}^{s'_0} ds' \rho_2^{K_1}(s, s', q^2) \theta[1 - (f^{K_1}(s, s'))^2] \right. \\
&\times \left. e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}} + \widehat{B} \Pi_{nonper2}^{K_1} \right], \tag{50}
\end{aligned}$$

where the  $\widehat{B} \Pi_{nonper}$  represents the double Borel transformation of the non-perturbative part in each case,  $s_0$  and  $s'_0$  are the continuum thresholds and the functions  $f^i(s, s')$  inside the step functions are determined requiring that the arguments of the three  $\delta$  functions coming from the Cutkosky rule vanish simultaneously. As a result, we find:

- $D(B)$  off-shell:

$$f^{D(B)}(s, s') = \frac{2 s (m_s^2 - m_u^2 + s') + (m_{c(b)}^2 - m_s^2 - s)(-q^2 + s + s')}{\lambda^{1/2}(m_{c(b)}^2, m_s^2, s) \lambda^{1/2}(s, s', q^2)}, \tag{51}$$

- $K_0^*(K)(K_1)$  off-shell:

$$f_1^{K_0^*(K)(K_1)}(s, s') = \frac{2 s (-m_{c(b)}^2 + m_u^2 - s') + (m_{c(b)}^2 - m_s^2 + s)(-q^2 + s + s')}{\lambda^{1/2}(m_{c(b)}^2, m_s^2, s) \lambda^{1/2}(s, s', q^2)}.$$

(52)

Here, we should stress that the physical regions are imposed by the limits on the integrals and step functions in the integrands in the sum rules expressions. In order to subtract the contributions of the higher states and continuum, the quark-hadron duality assumption in the following form is used:

$$\rho^{higherstates}(s, s') = \rho^{OPE}(s, s')\theta(s - s_0)\theta(s' - s'_0). \quad (53)$$

The double Borel transformation used in calculations is also defined in the following way:

$$\hat{B} \frac{1}{(p^2 - m_1^2)^m} \frac{1}{(p'^2 - m_2^2)^n} \rightarrow (-1)^{m+n} \frac{1}{\Gamma(m)} \frac{1}{\Gamma(n)} e^{-m_1^2/M^2} e^{-m_2^2/M'^2} \frac{1}{(M^2)^{m-1} (M'^2)^{n-1}}. \quad (54)$$

At the end of this section, we would like to mention that using Eqs. (1) and (16), the couplings to  $K_1(1270)$  and  $K_1(1400)$  are obtained in terms of the couplings to the  $K_1^{a(b)}$  as:

$$\begin{aligned} g_{D_s^* DK_1(1270)(B_s^* BK_1(1270))} &= g_{D_s^* DK_1^a(B_s^* BK_1^a)} \sin \theta + g_{D_s^* DK_1^b(B_s^* BK_1^b)} \cos \theta \\ g_{D_s^* DK_1(1400)(B_s^* BK_1(1400))} &= g_{D_s^* DK_1^a(B_s^* BK_1^a)} \cos \theta - g_{D_s^* DK_1^b(B_s^* BK_1^b)} \sin \theta \end{aligned} \quad (55)$$

### III. NUMERICAL ANALYSIS

In the present section, we numerically analyze the expressions of QCD sum rules obtained for the considered strong coupling constants. Some input parameters used in the calculations are:  $m_K = (493.677 \pm 0.016) \text{ MeV}$ ,  $m_{K_0^*}(800) = (672 \pm 40) \text{ MeV}$ ,  $m_{K_0^*}(1430) = (1425 \pm 50) \text{ MeV}$ ,  $m_{K_1}(1270) = (1272 \pm 7) \text{ MeV}$ ,  $m_{K_1}(1400) = (1403 \pm 7) \text{ MeV}$ ,  $m_D = (1.8648 \pm 0.00014) \text{ GeV}$ ,  $m_B = (5.2792 \pm 0.0003) \text{ GeV}$ ,  $m_{D_s} = (1.96847 \pm 0.00033) \text{ MeV}$ ,  $m_{B_s} = (5.3663 \pm 0.0006) \text{ MeV}$ ,  $m_{D_s^*} = (2.1123 \pm 0.0005) \text{ GeV}$ ,  $m_{B_s^*} = (5.4154 \pm 0.0014) \text{ GeV}$  [42],  $m_c = 1.3 \text{ GeV}$ ,  $m_b = 4.7 \text{ GeV}$ ,  $m_s = 0.14 \text{ GeV}$  [43],  $f_K = 160 \text{ MeV}$  [44],  $f_{K_0^*(800)}(1 \text{ GeV}) = (340 \pm 20) \text{ MeV}$ ,  $f_{K_0^*(1430)}(1 \text{ GeV}) = (445 \pm 50) \text{ MeV}$  [45],  $f_{D_s^*} = (272 \pm 16_{-20}^0) \text{ MeV}$ ,  $f_{B_s^*} = (229 \pm 20_{-16}^{31}) \text{ MeV}$  [46],  $f_B = 190 \pm 13 \text{ MeV}$  [47],  $f_D = (202 \pm 41 \pm 17) \text{ MeV}$  [48],  $f_{D_s} = (286 \pm 44 \pm 41) \text{ MeV}$  [49],  $f_{B_s} = 196 \text{ MeV}$  [50],  $\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle = -0.8(0.24 \pm 0.01)^3 \text{ GeV}^3$  [43],  $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$  [51],  $m_{K_1^a} = 1.31 \text{ GeV}$ ,  $m_{K_1^b} = 1.34 \text{ GeV}$ ,  $f_{K_1^a}(1 \text{ GeV}) = 0.25 \text{ GeV}$ ,  $f_{K_1^{b\perp}}(1 \text{ GeV}) = 0.19 \text{ GeV}$ ,  $a_0^{\parallel, K_1^b}(1 \text{ GeV}) = -0.19 \pm 0.07$  and  $a_0^{\perp, K_1^A}(1 \text{ GeV}) = 0.27_{-0.17}^{+0.03}$  [15, 20, 40].

The sum rule for strong coupling constants also contain four auxiliary parameters, namely the continuum thresholds  $s_0$  and  $s'_0$  related to the initial and final channels, respectively as well as Borel mass parameters  $M^2$  and  $M'^2$ . These quantities are mathematical objects, so according to the general criteria and standard procedure in QCD sum rules, our physical results should be insensitive to them. Therefore, we shall look for working regions of these quantities at which the dependence of coupling constants on these auxiliary parameters are weak. The working regions for the Borel mass parameters  $M^2$  and  $M'^2$  are determined demanding that both the contributions of the higher states and continuum are sufficiently suppressed and the contributions coming from the higher dimensional operators are small. Our calculations lead to the following working regions common for all cases:

- $D_s^{(*)} DK_0^*(K)(K_1)$  vertex:

- $D$  off-shell:  $8\text{GeV}^2 \leq M^2 \leq 25\text{GeV}^2$  and  $5\text{GeV}^2 \leq M'^2 \leq 15\text{GeV}^2$ ,
- $K_0^*(K)(K_1)$  off-shell:  $6\text{GeV}^2 \leq M^2 \leq 15\text{GeV}^2$  and  $4\text{GeV}^2 \leq M'^2 \leq 12\text{GeV}^2$ ,
- $B_s^{(*)}BK_0^*(K)(K_1)$  vertex:
  - $B$  off-shell:  $14\text{GeV}^2 \leq M^2 \leq 30\text{GeV}^2$  and  $5\text{GeV}^2 \leq M'^2 \leq 20\text{GeV}^2$ ,
  - $K_0^*(K)K_1$  off-shell:  $6\text{GeV}^2 \leq M^2 \leq 20\text{GeV}^2$  and  $5\text{GeV}^2 \leq M'^2 \leq 15\text{GeV}^2$ .

The continuum thresholds,  $s_0$  and  $s'_0$  are not completely arbitrary but they are correlated to the energy of the first excited states with the same quantum numbers as the considered interpolating currents. Our numerical calculations show that in the regions  $(m_i + 0.3)^2 \leq s_0 \leq (m_i + 0.7)^2$  and  $(m_f + 0.3)^2 \leq s'_0 \leq (m_f + 0.7)^2$ , respectively for the continuum thresholds  $s$  and  $s'$ , our results have weak dependence on these parameters. Here,  $m_i$  is the mass of initial particle and the  $m_f$  stands for the mass of the final on-shell state. For instance consider the  $g_{D_s DK_0^*}^D$  coupling constant at which the  $D$  meson is off-shell. This coupling constant describe the strong transition  $D_s \rightarrow DK_0^*$ , and for this case  $m_i = m_{D_s}$  and  $m_f = m_{K_0^*}$ .

As an example, we present the dependence of strong coupling constant  $g_{D_s DK_0^*}^{(D)}(800)$  on Borel mass parameters at  $Q^2 = 1 \text{ GeV}^2$ , where  $Q^2 = -q^2$  in Fig. (2). This figure demonstrates a good stability of the results with respect to the variations of Borel mass parameters in their working regions.

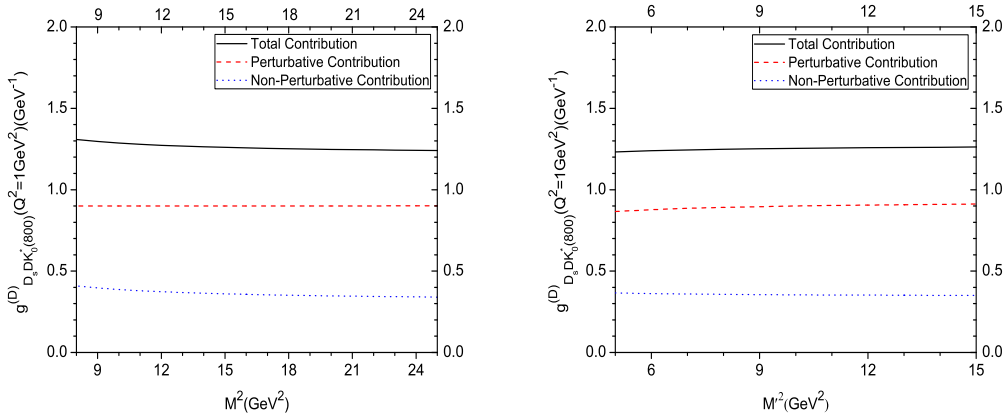


FIG. 2. **Left:**  $g_{D_s DK_0^*}^{(D)}(800)(Q^2 = 1 \text{ GeV}^2)$  as a function of the Borel mass  $M^2$  with  $M'^2 = 10 \text{ GeV}^2$ . **Right:**  $g_{D_s DK_0^*}^{(D)}(800)(Q^2 = 1 \text{ GeV}^2)$  as a function of the Borel mass  $M'^2$  with  $M^2 = 17 \text{ GeV}^2$ . The continuum thresholds  $s_0 = 6.09 \text{ GeV}^2$  and  $s'_0 = 1.37 \text{ GeV}^2$  have been used.

Now, we proceed to find the  $Q^2$  behavior of the considered strong coupling constants using the working regions for auxiliary parameters. First, we consider the scalar kaon case for both  $K_0^*(800)$  and  $K_0^*(1430)$ . The strong coupling constant in this case obeys from the following Boltzmann function:

$$g(Q^2) = A_1 + \frac{A_2}{1 + \exp\left[\frac{Q^2 - x_0}{\Delta x}\right]} [\text{GeV}^{-1}]. \quad (56)$$

The values of the parameters  $A_1$ ,  $A_2$ ,  $x_0$  and  $\Delta x$  for the considered coupling constant form factors are given in Table I. The coupling constants are defined as the values of the form

	$A_1$	$A_2$	$x_0$	$\Delta x$
$g_{D_s D K_0^*(800)}^{(D)}(Q^2)$	3.468	-2.741	8.067	4.995
$g_{D_s D K_0^*(800)}^{(K_0^*(800))}(Q^2)$	-0.024	0.772	5.723	1.257
$g_{D_s D K_0^*(1430)}^{(D)}(Q^2)$	4.712	-3.818	24.863	10.985
$g_{D_s D K_0^*(1430)}^{(K_0^*(1430))}(Q^2)$	-0.022	0.772	4.729	1.637
$g_{B_s B K_0^*(800)}^{(B)}(Q^2)$	4.151	-1.932	13.842	12.149
$g_{B_s B K_0^*(800)}^{(K_0^*(800))}(Q^2)$	-0.017	0.547	5.431	1.121
$g_{B_s B K_0^*(1430)}^{(B)}(Q^2)$	2.055	-0.207	11.239	5.084
$g_{B_s B K_0^*(1430)}^{(K_0^*(1430))}(Q^2)$	-0.004	0.255	4.819	1.146

TABLE I. Parameters appearing in the fit function of the coupling constants for  $D_s D K_0^*(800)$ ,  $D_s D K_0^*(1430)$ ,  $B_s B K_0^*(800)$  and  $B_s B K_0^*(1430)$  vertices.  $A_1$  and  $A_2$  are in  $GeV^{-1}$  units, while  $x_0$  and  $\Delta x$  are in the units of  $GeV^2$ .

factors at  $Q^2 = -m_{meson}^2$ , where  $m_{meson}$  is the mass of off-shell meson. The results of the coupling constants obtained using  $Q^2 = -m_{meson}^2$  are given in Tables II and III. The final result for each coupling constant is obtained taking the average of the coupling constants obtained from two different off-shell cases. The errors in the numerical results are due to the uncertainties in determination of the working regions for the auxiliary parameters as well as the errors in the input parameters.

	$Q^2 = -m_D^2$	$Q^2 = -m_{K_0^*(800)}^2$	Average
$g_{D_s D K_0^*(800)}$	$0.97 \pm 0.02$	$0.74 \pm 0.05$	$0.85 \pm 0.08$
	$Q^2 = -m_D^2$	$Q^2 = -m_{K_0^*(1430)}^2$	Average
$g_{D_s D K_0^*(1430)}$	$1.16 \pm 0.12$	$0.49 \pm 0.07$	$0.83 \pm 0.09$

TABLE II. Value of the  $g_{D_s D K_0^*(800,1430)}$  coupling constant in  $GeV^{-1}$  unit.

In the case of pseudoscalar kaon and  $D$  off-shell, the strong coupling constant is well described by the following monopolar fit parametrization:

$$g_{D_s^* D K}^{(D)}(Q^2) = \frac{8.76 (GeV^2)}{Q^2 + 7.12 (GeV^2)}, \quad (57)$$

The value of coupling constant obtained at  $Q^2 = -m_{meson}^2$  is presented in Table (IV).



	$Q^2 = -m_B^2$	$Q^2 = -m_{K_0^*(800)}^2$	Average
$g_{B_s B K_0^*(800)}$	$2.28 \pm 0.18$	$0.53 \pm 0.09$	$1.41 \pm 0.21$
	$Q^2 = -m_B^2$	$Q^2 = -m_{K_0^*(1430)}^2$	Average
$g_{B_s B K_0^*(1430)}$	$1.85 \pm 0.53$	$0.25 \pm 0.04$	$1.05 \pm 0.32$

TABLE III. Value of the  $g_{B_s B K_0^*(800,1430)}$  coupling constant in  $GeV^{-1}$  unit.

	$Q^2 = -m_D^2$	$Q^2 = -m_K^2$	Average
$g_{D_s^* DK}$ (Present work)	$2.79 \pm 0.24$	$2.99 \pm 0.26$	$2.89 \pm 0.25$
$g_{D_s^* DK}$ ([30])	2.72	2.87	$2.84 \pm 0.31$

TABLE IV. Value of the  $g_{D_s^* DK}$  coupling constant.

The result for strong coupling constant of pseudoscalar case and an off-shell  $K$  meson can be well fitted by the exponential parametrization

$$g_{D_s^* DK}^{(K)}(Q^2) = 3.55 e^{-\frac{Q^2}{7.25 (GeV^2)}} - 0.88, \quad (58)$$

where using  $Q^2 = -m_K^2$ , we obtain the result as also presented in the Table (IV). We also depict the final result for this case taking the average of two above obtained values. This Table also shows the predictions of [30] on  $g_{D_s^* DK}$  as the only existing previously calculated coupling constant among the considered vertices. Comparing our results with that of [30], we see a good consistency between two predictions.

Similarly, for  $B_s^* BK$  vertex, our result for the pseudoscalar kaon and  $B$  off-shell is better extrapolated by the exponential fit parametrization

$$g_{B_s^* BK}^{(B)}(Q^2) = 0.66 e^{-\frac{Q^2}{23.34 (GeV^2)}} + 0.23, \quad (59)$$

and in the  $K$  off-shell case by the parametrization

$$g_{B_s^* BK}^{(K)}(Q^2) = 4.39 e^{-\frac{Q^2}{4.02 (GeV^2)}} - 1.03. \quad (60)$$

Using the same procedure as above, we find the values depicted in the Table (V).

In the case of axial vector kaon, the strong coupling constant obey also the same Boltzmann function as the scalar case. The values of the parameters  $A_1$ ,  $A_2$ ,  $x_0$  and  $\Delta x$  for coupling constants in this case are given in Table VI. The same procedure as in the scalar and pseudoscalar cases leads to the numerical results for the corresponding coupling constants as presented in the Tables (VII and VIII).

In summary, the strong coupling constants,  $g_{B_s B K_0^*}$ ,  $g_{D_s DK_0^*}$ ,  $g_{B_s^* BK}$ ,  $g_{D_s^* DK}$ ,  $g_{B_s^* BK_1}$  and  $g_{D_s^* DK_1}$ , have been calculated in the framework of three-point QCD sum rules. The correlation functions of the considered vertices when both  $B(D)$  and  $K_0^*(K)(K_1)$  mesons are

	$Q^2 = -m_B^2$	$Q^2 = -m_K^2$	Average
$g_{B_s^*BK}$	$2.40 \pm 0.22$	$3.62 \pm 0.34$	$3.01 \pm 0.28$

TABLE V. Value of the  $g_{B_s^*BK}$  coupling constant.

	$A_1$	$A_2$	$x_0$	$\Delta x$
$g_{D_s^*DK_1(1270)}^{(D)}(Q^2)$	5.062	-2.337	1.182	1.531
$g_{D_s^*DK_1(1400)}^{(D)}(Q^2)$	73.848	-87.162	118.101	74.590
$g_{D_s^*DK_1(1270)}^{(K_1(1270))}(Q^2)$	0.137	-1.507	6.951	1.845
$g_{D_s^*DK_1(1400)}^{(K_1(1400))}(Q^2)$	-0.106	1.234	6.843	1.847
$g_{B_s^*BK_1(1270)}^{(B)}(Q^2)$	0.764	0.412	11.343	4.708
$g_{B_s^*BK_1(1400)}^{(B)}(Q^2)$	2.463	-2.178	38.732	18.980
$g_{B_s^*BK_1(1270)}^{(K_1(1270))}(Q^2)$	0.047	-23681.595	-31.416	2.914
$g_{B_s^*BK_1(1400)}^{(K_1(1400))}(Q^2)$	-0.021	0.282	3.080	1.233

TABLE VI. Parameters appearing in the fit function of the coupling constants for  $D_s^*DK_1(1270)$ ,  $D_s^*DK_1(1400)$ ,  $B_s^*BK_1(1270)$  and  $B_s^*BK_1(1400)$  vertices.  $A_1$  and  $A_2$  are in  $GeV^{-1}$  units, while  $x_0$  and  $\Delta x$  are in the units of  $GeV^2$ .

off-shell are evaluated. The final numerical values have been obtained taking the average of the numerical values obtained from both off-shell cases. In the case of the axial vector  $K_1$ , which is either  $K_1(1270)$  or  $K_1(1400)$ , the mixing between these two states have also been taken into account. A comparison of the obtained result on  $D_s^*DK$  as the only previously calculated coupling constant among the considered strong coupling constants has also been made.

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	$Q^2 = -m_D^2$	$Q^2 = -m_{K_1(1270)}^2$	Average
$g_{D_s^* DK_1(1270)}$	$2.83 \pm 0.09$	$1.36 \pm 0.14$	$2.09 \pm 0.82$
	$Q^2 = -m_D^2$	$Q^2 = -m_{K_1(1400)}^2$	Average
$g_{D_s^* DK_1(1400)}$	$0.97 \pm 0.15$	$1.12 \pm 0.54$	$1.04 \pm 0.78$

TABLE VII. Values of the  $g_{D_s^* DK_1(1270)}$  and  $g_{D_s^* DK_1(1400)}$  coupling constants in  $GeV^{-1}$ .

	$Q^2 = -m_B^2$	$Q^2 = -m_{K_1(1270)}^2$	Average
$g_{B_s^* BK_1(1270)}$	$1.18 \pm 0.07$	$0.81 \pm 0.45$	$1.99 \pm 0.11$
	$Q^2 = -m_B^2$	$Q^2 = -m_{K_1(1400)}^2$	Average
$g_{B_s^* BK_1(1400)}$	$0.35 \pm 0.05$	$0.26 \pm 0.04$	$0.30 \pm 0.05$

TABLE VIII. Values of the  $g_{B_s^* BK_1(1270)}$  and  $g_{B_s^* BK_1(1400)}$  coupling constants in  $GeV^{-1}$ .

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